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Effects of the confining interaction on mesoscopic superconducting cylinders

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During the last few years, interest in studying the properties of mesoscopic superconductors has considerably increased, due to advances in nanotechnology. One of the most important features of such systems is the considerable consequence of sample size and form on the superconducting properties.

Traditionally these effects are studied by means of the Ginzburg-Landau (GL) equations, under a boundary condition whose normal component at the sample surface vanishes on the superconducting current [1–3], which implies that there are no carriers moving outside the sample. In the theory developed for macroscopic samples, the free-energy GL functional does not include the contribution to the energy of Cooper pairs due to the confinement interaction, because of its marginal effect.

On the other hand, for mesoscopic samples, the contribution caused by the confinement interaction is not negligible because the sample size is comparable to the pair coherence length, and therefore it is of interest to develop a theory considering such contributions. In a recent work, Shanenko et al. [4, 5], generalized the GL theory in such a way that the terms which originated from the confinement interaction appear explicitly in the GL equation for the superconducting order parameter.

It is the aim of the present communication to apply this model in order to study the thermodynamic critical field in samples with cylindrical symmetry with simple connectivity (bulk cylinder) and with nontrivial connectivity (superconductor shell). Special attention will be devoted to a comparison with results obtained with other geometries [4, 5].

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Let us consider a superconductor sample with cylindrical symmetry of height l, internal radius a and external radius b, such that it is satisfies the condition $(l, a, b) \geq \xi(T) \geq \xi_0$, where $\xi(T)$ is the temperature dependent coherence length and $\xi_0 = \xi(0)$. When the confining interaction is taken into account, in the absence of applied magnetic field, the superconducting state is described by the nonhomogeneous order parameter $\Psi(r)$, which satisfies the equation [3]:

$$\left[-\frac{\hbar^2}{2m^*}\nabla^2 + 2V_{conf}(\boldsymbol{r}) + \alpha + \beta\ |\Psi(\boldsymbol{r})|^2\right]\Psi(\boldsymbol{r}) = 0\,, \tag{1}$$

which is formally identical to the Gross-Pitaevskii equation [7, 8].

The order parameter $\Psi(\mathbf{r})$ must satisfy the normalization condition:

$$n = \frac{1}{\Omega}\int d^3r\ |\Psi(\boldsymbol{r})|^2\,, \tag{2}$$

Ω being the sample volume, and n the mean density of Cooper pairs.

We assume additionally that the confinement potential has the form $V_{conf} = 0$ (if $a < \rho < b$, $0 < z < l$) and infinity otherwise, which implies that at the sample boundaries the order parameter vanishes ($\Psi(\mathbf{r})|_{boundary} = 0$). For a sample with axial symmetry the order parameter $\Psi(\mathbf{r})$ satisfies the equation

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Psi}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2\Psi}{\partial\varphi^2} + \frac{\partial^2\Psi}{\partial z^2} - \frac{2m^*}{\hbar^2}\alpha_c\psi = 0\,, \tag{3}$$

under the boundary conditions $\Psi(\rho = a, z) = \Psi(\rho = b, z) = \Psi(\rho, z = 0) = \Psi(\rho, z = l) = 0$.

The solution of this boundary value problem is

$$\Psi(\rho, z) = \sqrt{n(1-f^2)}\frac{\left[J_0(xr) - \frac{J_0(x)}{N_0(x)}N_0(xr)\right]}{\sqrt{\int_f^1\left[J_0(xr) - \frac{J_0(x)}{N_0(x)}N_0(xr)\right]^2 rdr}}\sin\left(\frac{\pi}{l}z\right), \tag{4}$$

where $r = \rho/b$, f is the relation between the inner and outer radii ($f = a/b$), J_0 and N_0 are the Bessel and Neumann functions [9], respectively, and x are the solutions of the non linear equation

$$J_0(xf)N_0(x) - J_0(x)N_0(xf) = 0\,, \tag{5}$$

For the particular case of a bulk cylinder ($a = 0$, $0 < \rho < b$, $0 < z < l$) the order parameter takes the form

$$\Psi(\rho, z) = \frac{\sqrt{n}}{0.3678}J_0\left(\frac{x_0}{b}\rho\right)\sin\left(\frac{\pi}{l}z\right), \tag{6}$$

where $x_0 \approx 2.40483$ is the lowest zero of Bessel function.

The thermodynamic critical field H_c can be calculated from the expression

$$H_c^2 = \frac{4\pi\beta}{\Omega}\int d^3r|\Psi(\boldsymbol{r})|^4\,, \tag{7}$$

which follows from the difference between the free energies of the normal and the superconductor states of the mesoscopic sample.

For the considered mesoscopic system we obtain

$$H_c^2 = 3\pi \quad \beta \quad n^2\left(1-f^2\right)\frac{\int_f^1 \left[J_0(xr)-\frac{J_0(x)}{N_0(x)}N_0(xr)\right]^4 rdr}{\left\{\int_f^1 \left[J_0(xr)-\frac{J_0(x)}{N_0(x)}N_0(xr)\right]^2 rdr\right\}^2} . \tag{8}$$

By measuring the magnetic fields in units of $\sqrt{\beta n}$, we have the following result:

$$\frac{H_c}{\sqrt{\beta n}} = \sqrt{3\pi\left(1-f^2\right)}\frac{\sqrt{\int_f^1 \left[J_0(xr)-\frac{J_0(x)}{N_0(x)}N_0(xr)\right]^4 rdr}}{\int_f^1 \left[J_0(xr)-\frac{J_0(x)}{N_0(x)}N_0(xr)\right]^2 rdr} . \tag{9}$$

For the particular case of a bulk cylinder, the latter expression has the form:

$$\frac{H_c}{\sqrt{\beta n}} = \sqrt{3\pi}\sqrt{\frac{\int_0^1 [J_0(x_0 r)]^4 rdr}{\left[\int_0^1 [J_0(x_0 r)]^2 rdr\right]^2}} \approx 6.29 . \tag{10}$$

For non mesoscopic samples, whose dimensions are much greater than the coherence length, the confinement interaction can be neglected and in such a case the corresponding thermodynamic critical field H_c^0 is obtained from the definition $\left(H_c^0\right)^2 = 4\pi \quad \left(\alpha^2/\beta\right)$, which gives

$$\frac{H_c^0}{\sqrt{\beta n}} = \sqrt{4\pi} \approx 3.54 . \tag{11}$$

Therefore, the relation between the thermodynamic critical field of mesoscopic samples with axial symmetry and the corresponding field H_c^0 is given by

$$\frac{H_c}{H_c^0} \approx 0.87\sqrt{\left(1-f^2\right)}\frac{\sqrt{\int_f^1 \left[J_0(xr)-\frac{J_0(x)}{N_0(x)}N_0(xr)\right]^4 rdr}}{\left[\int_f^1 \left[J_0(xr)-\frac{J_0(x)}{N_0(x)}N_0(xr)\right]^2 rdr\right]} . \tag{12}$$

For the particular case of a mesoscopic cylinder, we obtain:

$$\frac{H_c}{H_c^0} \approx 1.77 . \tag{13}$$

This result means that the confinement potential in mesoscopic samples enhances the value of the thermodynamic critical field in comparison with the value obtained for samples where the coherence length of the Cooper pairs is negligible compared with the sample size. Such an effect of enhancement has been studied theoretically for samples of spherical and cubic form [4] and is in accordance with the increment in a factor of 4-5 observed experimentally for the upper critical field H_{c2} in mesoscopic samples [10].

In Table 1 we compare our results concerning a cylindrical sample with those obtained by Shanenko-Ivanov [4] for samples of spherical and cubic form. We can see that the value of the critical field corresponding to the cylindrical sample lies between the values of the spherical and the cubic samples. This can be understood if we take into account that the number of elements of the symmetry group of a system with axial symmetry is greater than those of the cube but lower than of the sphere, and the critical field rises with the number of such elements.

Table 1 Thermodynamic critical fields for samples with different geometries.

Cubic sample [4]	Cylindrical sample	Spherical sample [4]
$H_c = 6.51n\sqrt{\beta}$	$H_c = 6.29n\sqrt{\beta}$	$H_c = 5.95n\sqrt{\beta}$
$\frac{H_c}{H_c^0} = 1.83$	$\frac{H_c}{H_c^0} = 1.77$	$\frac{H_c}{H_c^0} = 1.68$

In Fig. 1 we have plotted the behavior of the thermodynamic critical field H_c (in units of $n\sqrt{\beta}$) of the considered cylindrical shell sample as a function of the relation f between the inner and outer radii.

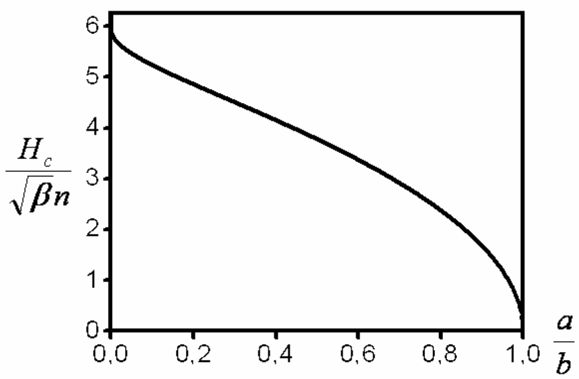

Fig. 1 Thermodynamic critical field H_C (in units of $n\sqrt{\beta}$) as a function of the relation between the inner and the outer radii of a sample with axial symmetry.

We observe that the thermodynamic critical field diminishes when the inner radius a approach the outer radius and tends abruptly to zero when $a = b$. In order to study this regime with more accuracy, a more realistic confinement potentials should be taken into account. Additionally, an abrupt change in H_c is observed close to $a = 0$. This is due to the concomitant abrupt change in the topological properties of the sample: for $a = 0$ we have a system with a single connectivity, and for $a \neq 0$ we have a double connected sample, whose non trivial topological properties lead to new features in the properties of superconductors [11].

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